Teacher Notes – KEY

CW#71H: Application/Review 10.13-10.16
Honors Geometry

CRS	FUN 703 - Exhibit knowledge of unit circle trigonometry.
Objective	10.13 – Find exact values of the 6 trigonometric functions(4) 10.14 – Find reference angles(2) 10.15A – Determine the quadrant [including naming quadrants as an interval] of an angle given the sign(s) of trig function(s) (4) 10.15B - Evaluate the 6 trigonometric functions of any angle using reference angles 10.16 – Evaluate inverse trig functions (4)

Homework Review:

1) Solve the equation for θ without a calculator. Give your answer in both radians and degrees. $\tan\theta = (-\sqrt{3})$	2)) Solve the equation for θ without a calculator. Give your answer in both radians and degrees. $\cos\theta = (-\frac{1}{2})$	3) Solve the equation for θ without a calculator. Give your answer in both radians and degrees. $\sin\theta = (-\frac{\sqrt{3}}{2})$

Quotient Identities:

$$\tan\ \theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\ \theta = \frac{\cos\theta}{\sin\theta}$$

Ex. Prove the cotangent identity.

Pythagorean Identities: $\sin^2\theta + \cos^2\theta = 1$

Ex. $(\text{Sin } 45^\circ)^2 + (\text{Cos } 45^\circ)^2 = 1$ Prove your answer.

Reciprocal Identities: $\csc\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

Be able to do the same with these ratios.

Name: Key ◡̈ TP: ________

CW#71H: Application/Review 10.13-10.16
Honors Geometry

CRS	FUN 703 - Exhibit knowledge of unit circle trigonometry.
Objective	10.13 – Find exact values of the 6 trigonometric functions(4) 10.14 – Find reference angles(2) 10.15A – Determine the quadrant [including naming quadrants as an interval] of an angle given the sign(s) of trig function(s) (4) 10.15B - Evaluate the 6 trigonometric functions of any angle using reference angles 10.16 – Evaluate inverse trig functions (4)

Homework Review:

1) Solve the equation for θ without a calculator. Give your answer in both radians and degrees.
$\tan\theta = (-\sqrt{3})$

$\tan^{-1}\theta =$
$-90° < \theta < 90°$
↑ not equal to b/c θ is undefined at 90!
$\theta = -60°$

2)) Solve the equation for θ without a calculator. Give your answer in both radians and degrees.
$\cos\theta = (-\frac{1}{2})$

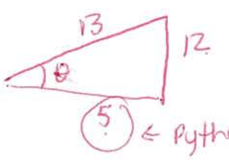
$\cos^{-1}$ domain
$0° \le \theta \le 180°$
$\theta = 120°$

3) Solve the equation for θ without a calculator. Give your answer in both radians and degrees.
$\sin\theta = (-\frac{\sqrt{3}}{2})$

$\sin^{-1}\theta =$
$-90° \le \theta \le 90°$
$\theta = -60°$

Objective 10.13: **Find the <u>exact</u> value of each of the remaining trigonometric functions of θ.**

1. $\sin\theta = \frac{12}{13}$

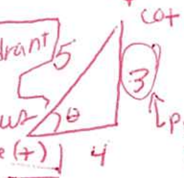


$\csc\theta = 13/12$
$\sec\theta = 13/5$
$\cos\theta = 5/13$
$\cot\theta = 5/12$
$\tan\theta = 12/5$

2. $\cos\theta = -\frac{4}{5}$ → if $\cos\theta < 0$ then $\sec\theta < 0$ & $\tan < 0$ b/c $\tan\theta = \frac{\sin\theta}{\cos\theta}$ ↓ $\cot\theta < 0$

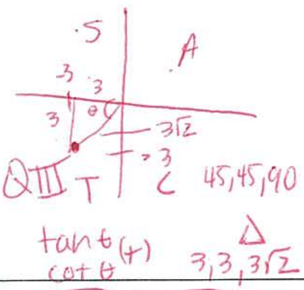
$\cos\theta = -4/5$ → $\sec\theta = -5/4$
$\sin\theta = 3/5$
$\csc\theta = 5/3$
$\tan\theta = -3/4$
$\cot\theta = -4/3$

*there is no quadrant where all trig funct are (-) thus sin must be (+)



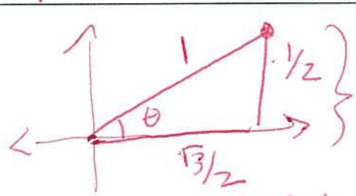
3. (-3, -3)

$\sin\theta = \frac{3}{3\sqrt{2}} = \frac{-\sqrt{2}}{2}$
$\cos\theta = \frac{-\sqrt{2}}{2}$
$\tan\theta = \frac{-3}{-3} = 1$
$\cot\theta = 1$
$\sec\theta = \frac{-3\sqrt{2}}{3} = -\sqrt{2}$
$\csc\theta = -\sqrt{2}$



* Notice these are the same values for a 45° Δ in quad 1 .... Hmm...

4. $(\frac{\sqrt{3}}{2}, \frac{1}{2})$
x y



} Unit circle values in Q1

$\sin\theta = \frac{1}{2}$
$\cos\theta = \sqrt{3}/2$
$\tan\theta = \sqrt{3}/3$
$\sec\theta = 2\sqrt{3}/3$
$\csc\theta = 2$
$\cot\theta = \sqrt{3}$

all are positive.

Objective: 10.14 – Find reference Angles

Find the reference angle of each angle. **GRAPH THE ANGLES IN THE COORDINATE PLANE.**

5. 150°

150°

30° = θ'

in QII

180 − θ = θ'

6. 120°

120°

θ'

in QII

180 − θ = θ'

θ' = 60°

7. Rewrite the expression using reference angles **and** then solve.

$\frac{17\pi}{6} * \frac{30\ 180}{\pi} = 510°$
-360
150
$\theta' = 180 - 150 = 30°$

sin 135° + cos 17π/6 + tan (-π/3)

$(\sin 45°) + \cos(30°) + (\tan -60°)$

$\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right) + \left(-\left(\frac{\sqrt{3}}{1}\right)\right)$

$= \boxed{\frac{\sqrt{2} - \sqrt{3}}{2}}$

2, 60°, 1, 30°, $\sqrt{3}$

8. $\frac{5\pi}{4}$

$\frac{5\pi}{4} * \frac{180}{\pi}$ 45

$4\overline{)180}$ 45

$\boxed{\theta' = 45°}$

225°

$5 * 45 = 225° = \frac{5\pi}{4}$

in QIII $\left\{ \begin{array}{l} \theta \\ 225 - 180 = \theta' \end{array} \right.$

Objective: 10.15a – Determine the quadrant [including naming quadrants as an interval] of an angle given the sign(s) of trig function(s) (4)

Directions: Name the **quadrants** for the following reference angles, given the inequalities.

Ref angle = the acute angle formed by θ and the x-axis.

9. If 180° < θ < 270°, then we know θ lies in which quadrant?

QIII

90, 180, 0, 360, 270

And the reference angle θ' = θ − 180°

10. If 270° < θ < 360°, then we know θ lies in which quadrant?

QIV

90, 180, 0, 360, 270

And the reference angle θ' = 360 − θ

11. If 90° < θ < 180°, then we know θ lies in which quadrant?

QII

90, 0, 360, 270

And the reference angle θ' = 180 − θ

12. Practice Quiz Question: Find the remaining 6 trig functions of θ given the following:

30, 60, 90 △

in QIV

(+) (−)

sec θ = 2 , sin θ < 0

hyp/adj

S A T C

2, θ, 1, $\sqrt{3}$ ← Pythag.

$\sin\theta = \sqrt{3}/2$ $\tan\theta = \sqrt{3}$ cos sec

$\cos\theta = 1/2$ $\cot\theta = \sqrt{3}/3$

Explain how you determined the quadrant of θ and include a graph of your triangle with sec θ = 2.

$2^2 = b^2 + 1^2 = 3^2 = b^2$ $\boxed{b = \sqrt{3}}$ $\boxed{\csc\theta = \frac{2\sqrt{3}}{3}}$

13. Practice Quiz Question: Find the exact value for the following:

We're in QII

sin 510°

2, 30°, 1, $\sqrt{3}$

#7. 510 = 30° ref △

$\boxed{\sin 30 = 1/2}$ ✓ sin is + in QII

Explain how you determined the quadrant of θ and include a graph of your triangle.

Objective: 10.15B - Evaluate the 6 trigonometric functions of any angle using reference angles

14. sin 510° = 1/2

$\cos\theta = \boxed{\cos 30 = -\sqrt{3}/2}$

in QII Repeat Question
sin and csc (+)

$\boxed{\tan\theta = -\frac{\sqrt{3}}{3}}$ $\boxed{\cot\theta = -\sqrt{3}}$ $\boxed{\sec\theta = -\frac{2\sqrt{3}}{3}}$

$\boxed{\csc\theta = 2}$

2, 30°, $\sqrt{3}$

15. cos 600°

Always graph the angle to see where its signs fall

S A T C

240°

60°

QIII

1

cos 60°

$\boxed{= -1/2}$

600
−360
240

2, 30°, 1, $\sqrt{3}$

$\frac{3\pi}{4} * \frac{180}{\pi} = 135^\circ$

16. cos 540° − tan (−405°)

$\cos(180) - (-\tan(45))$

$\cos(0) - (-1)$

$-1 + 1 = \boxed{0}$

$(-1, 0)$

17. $6\cos\left(\frac{3\pi}{4}\right) + 2\tan\left(-\frac{\pi}{3}\right)$

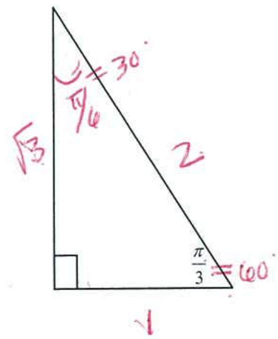
$\cos(45)$ $\quad -\tan(60^\circ)$

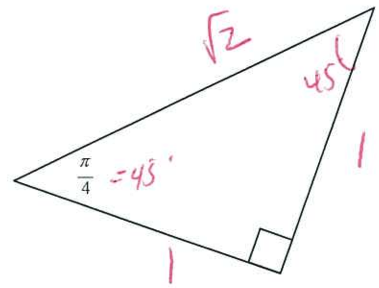
135° $\quad 6\left(\frac{-\sqrt{2}}{2}\right) + 2(-\sqrt{3})$

$-3\sqrt{2} + (-2\sqrt{3}) = \boxed{-3\sqrt{2} - 2\sqrt{3}}$

$\theta' = 45^\circ$ and cos (−)

Fill in the ratios for the special right triangles given the location of the marked angle. Use these ratios to help you answer 18-23





Objective: 10.6 Evaluate inverse trig functions (4)

18. $\cos^{-1}\frac{\sqrt{3}}{2} = \frac{adj}{hyp}$

using special right triangles

θ = 30 **in degrees** θ = π/6 **in radians**

19. $\tan^{-1}\frac{\sqrt{3}}{1} = \frac{opp}{adj}$

using special right triangles

θ = 60 **in radians** θ = π/3 **in degrees**

20. $\sec^{-1}\frac{\sqrt{2}}{1} = \frac{hyp}{adj}$

θ = 45 **in radians** θ = π/4 **in degree**

21. $\cot\theta = \frac{1}{1} = \frac{adj}{opp}$

$\theta = \cos^{-1}(1)$

θ= 45 **in radians** θ = π/4 **in degrees**

22. CHALLENGE **EXAMPLE:** Solve for x

$\sin^{-1}(x-1) = \frac{\pi}{4}$

➔ (To free the "x-1" I take the sin of both sides)

$(x-1) = \sin\frac{\pi}{4}$ (sub in the values I know)

$x = 1 + \frac{\sqrt{2}}{2}$ Easy enough!

23. CHALLENGE PROBLEM: You try!

Will not be on the QUIZ

Solve for x in the expression: $\tan^{-1}(x + 2) = 1$ ← angle

side length

$\tan(\tan^{-1}(x+2)) = \tan(1)$

$x + 2 = \tan(1)$
$-2 \qquad -2$

$x = \tan(1) - 2$

Rad $x \approx -.443$ or -1.98°